Chapter 12. Distance and Section Formulae

Ex 12.1

Answer 2.

Coordinates of origin are P (0, 0).

(a) $P(0,0)$, $Q(5,12)$

$$PQ=\sqrt{(12-0)^2+(5-0)^2}=\sqrt{144+25}=\sqrt{169}=13\text{units}$$

(b) $P(0,0)$, $Q(6,8)$

$$PQ=\sqrt{(6-0)^2+(8-0)^2}=\sqrt{36+64}=\sqrt{100}=10\text{units.}$$

(c) $P(0,0)$, $Q(8,15)$

$$PQ=\sqrt{(8-0)^2+(15-0)^2}=\sqrt{64+225}=\sqrt{289}=17\text{ units.}$$

(d) $P(0,0)$, $Q(0,11)$

$$PQ=\sqrt{(0-0)^2+(11-0)^2}=\sqrt{121}=11\text{units}$$

(e) $P(0,0)$, $Q(13,0)$

$$PQ=\sqrt{(13-0)^2+(0-0)^2}=\sqrt{169}=13\text{units}$$

Answer 3.

(a) A (p+q, p-q),   B (p-q, p-q)

$$AB=\sqrt{(p-q-p)^2+(p-q-p+q)^2}$$

$$=\sqrt{4q^2+0}=2q\text{units}$$

(b) $A(\sin\theta, \cos\theta)$, $B(\cos\theta, -\sin g\theta)$

$$AB=\sqrt{(\cos\theta-\sin\theta)^2+(-\sin\theta-\cos\theta)^2}$$

$$=\sqrt{\cos^2\theta+\sin^2\theta-2\cos\theta\sin\theta+\sin^2\theta+\cos^2\theta+2\cos\theta.\sin\theta}$$

$$=\sqrt{2}\text{units.}$$

(c) $A(\sec\theta, \tan\theta)$, $B(-\tan\theta, \sec\theta)$

$$AB=\sqrt{(-\tan\theta-\sec\theta)^2+(\sec\theta-\tan\theta)^2}$$

$$=\sqrt{\tan^2\theta+\sec^2\theta+2\tan\theta\sec\theta+\sec^2\theta+\tan^2\theta-2\tan\theta.\sec\theta}$$

$$=\sqrt{2\sec^2\theta+2\tan^2\theta}\text{units.}$$

) $A(\sin\theta-\operatorname{cosec}\theta, \cos\theta-\cot\theta)$

$B(\cos\theta-\operatorname{cosec}\theta, -\sin\theta-\cot\theta)$

$$AB=\sqrt{(\cos\theta-\operatorname{cosec}\theta-\sin\theta+\operatorname{cosec}\theta)^2+(-\sin\theta-\cot\theta-\cos\theta+\cot\theta)^2}$$

$$=\sqrt{(\cos\theta-\sin\theta)^2+(-\sin\theta-\cos\theta)^2}$$

$$=\sqrt{\cos^2\theta+\sin^2\theta-2\cos\theta\sin\theta+\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta}$$

$$=\sqrt{2}\text{units.}$$

Answer 4.

Let the point on x-axis be (x, 0) given abscissa is -5.

$\therefore$ point is $P(-5,0)$

Let $(7,5)$ be point A

$$AP=\sqrt{(7+5)^2+(5-0)^2}$$

$$=\sqrt{144+25}$$

$$=\sqrt{169}$$

$$=13\text{units}$$

Answer 5.

Point on the line y = 0 lies on x-axis given absdissa is 1.

$\therefore$point is $P(1, 0)$

Let$(13, -9)$be point A

$$AP = \sqrt{(13-1)^2 + (-9-0)^2}$$
$$= \sqrt{144+81}$$
$$= \sqrt{225}$$
$$= 15 \text{units}$$

Answer 6.

Point on the line x = 0 lies on given its ordinate is 9.

$\therefore$point is $P(0,9)$

Let the point $(12,5)$ be A.

$$AP = \sqrt{(12-0)^2 + (5-9)^2}$$
$$= \sqrt{144+16}$$
$$= \sqrt{160}$$
$$= 4\sqrt{10} \text{units.}$$

Answer 7.

Let the points (5, a) and (1, -5) be P and Q respectively.

Given, PQ = 5 units

$$\sqrt{(5-1)^2 + (a+5)^2} = 5$$

squaring both sides, we get,

$$16 + a^2 + 25 + 10a = 25$$
$$\Rightarrow a^2 + 10a + 16 = 0$$
$$\Rightarrow a^2 + 8a + 2a + 16 = 0$$
$$\Rightarrow (a+8)(a+2) = 0$$
$$\therefore a = -8, -2$$

Answer 8.

Let the points (m, -4) and (3, 2) be A and B respectively.

Given AB $= 3\sqrt{5}$ units

$\sqrt{(m-3)^2+(-4-2)^2} = 3\sqrt{5}$

squaring both sides

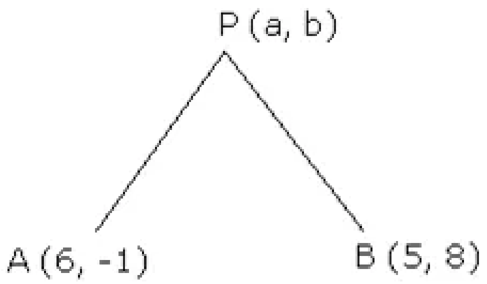
$m^2 - 6m + 9 + 36 = 45$

$\Rightarrow m^2 - 6m = 0$

$\Rightarrow m(m-6) = 0$

$\Rightarrow m = 0$ or 6.

Answer 9.



Given, PA = PB

$\therefore PA^2 = PB^2$

$\Rightarrow (a-6)^2 + (b+1)^2 = (a-5)^2 + (b-8)^2$

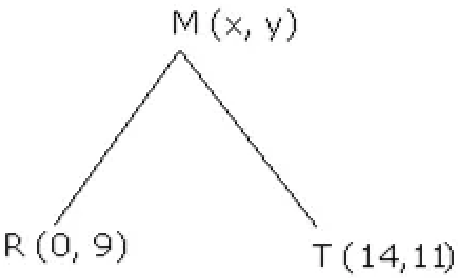
$\Rightarrow a^2 + 36 - 12a + b^2 + 1 + 2b = a^2 + 25 - 10a + b^2 + 64 - 16b$

$\Rightarrow -2a + 18b - 52 = 0$

$\Rightarrow -a + 9b - 26 = 0$

$\Rightarrow a = 9b - 26$

Answer 10.



Given: MR = MT

$\therefore MR^2 = MT^2$

$(x-0)^2 + (y-9)^2 = (x-14)^2 + (y-11)^2$

$x^2 + y^2 + 81 - 18y = x^2 + 196 - 28x + y^2 + 121 - 22y$

$81 - 18y = 196 - 28x + 121 - 22y$

$28x - 18y + 22y = 196 + 121 - 81$

$28x + 4y = 236$

$7x + y - 58 = 0$

Answer 11.

P lies on y-axis and has ordinate

$\therefore P(0,5)$

Qlieson x − axis and has an abscissa

$\therefore Q(12,0)$

$\therefore PQ = \sqrt{(12-0)^2 + (0-5)^2}$

$= \sqrt{144+25}$

$= \sqrt{169}$

$= 13$units.

Answer 12.

P lies on x-axis and Q lies on y-axis
Let abscissa of P be x then ordinate of Q is x-1.

$\therefore P(x, 0), \quad Q(0, x-1)$

Given PQ = 5 units

$\sqrt{(x-0)^2 + (0-x+1)^2} = 5$

squaring both sides

$x^2 + x^2 + 1 - 2x = 25$

$2x^2 - 2x - 24 = 0$

$x^2 - x - 12 = 0$

$x^2 - 4x + 3x - 12 = 0$

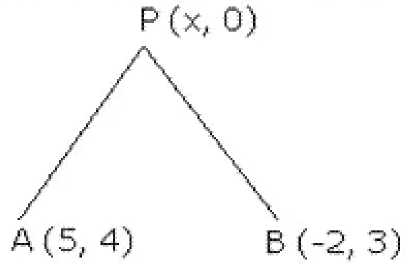
$(x-4)(x+3) = 0$

$x = +4 \text{ or } -3$

Coordinates of P are (4, 0) or (-3, 0)
Coordinates of Q are (0, 3) or (0, -4).

Answer 13.

Let the point on x-axis be P (x, 0).



Given,

$PA = PB$

$PA^2 = PB^2$

$(x-5)^2 + (0-4)^2 = (x+2)^2 + (0-3)^2$

$x^2 + 25 - 10x + 16 = x^2 + 4 + 4x + 9$

$\Rightarrow -14x + 28 = 0$

$\Rightarrow 14x = 28$

$\Rightarrow x = 2$

$\therefore$ The point on x – axis is $(2, 0)$

Answer 14.

A (-4, 3). Let the other point B (x, 9).

Given, AB = 10 units

$$\sqrt{(-4-x)^2+(3-9)^2}=10$$

squaring both sides,

$$\Rightarrow 16+x^2+8x+36=100$$

$$\Rightarrow x^2+8x-48=0$$

$$\Rightarrow x^2+12x-4x-48=0$$

$$\Rightarrow x(x+12)-4(x+12)=0$$

$$\Rightarrow (x-4)\ (x+12)=0$$

$$\Rightarrow x=4 \text{ or } -12$$

The abscissa of other end is 4 or -12.

Answer 15.

A (5, 5), B (3, 4), C (-7, -1)

$$AB=\sqrt{(5-3)^2+(5-4)^2}=\sqrt{4+1}=\sqrt{5}\text{ units}$$

$$BC=\sqrt{(3+7)^2(4+1)^2}=\sqrt{100+25}=5\sqrt{5}\text{ units}$$

$$AC=\sqrt{(5+7)^2+(5+1)^2}=\sqrt{144+36}=6\sqrt{5}\text{ units}$$

$$AB+BC=\sqrt{5}+5\sqrt{5}=6\sqrt{5}=AC$$

$\because AB+BC=AC$

ce, A, B and C are collinear points

P(5, 1), Q(3, 2), R(1, 3)

$$PQ=\sqrt{(5-3)^2+(1-2)^2}=\sqrt{4+1}=\sqrt{5}\text{ units}$$

$$QR=\sqrt{(3-1)^2+(2-3)^2}=\sqrt{4+1}=\sqrt{5}\text{ units}$$

$$PR=\sqrt{(5-1)^2+(1-3)^2}=\sqrt{16+4}=\sqrt{20}\text{ units}$$

$$PQ+QR=\sqrt{5}+\sqrt{5}=2\sqrt{5}=PR$$

$\because PQ+QR=PR$

$\therefore$ P, Q and R are collinear points

M(4, −5), N(1, 1), S(−2, 7)

$$MN=\sqrt{(4-1)^2+(-5-1)^2}=\sqrt{9+36}=3\sqrt{5}\text{ units}$$

$$NS=\sqrt{(1+2)^2+(1-7)^2}=\sqrt{9+36}=3\sqrt{5}\text{ units}$$

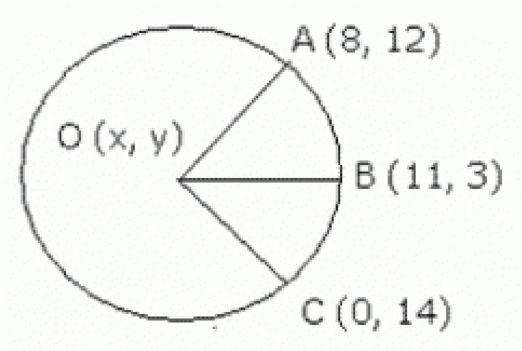
$$MS=\sqrt{(4+2)^2+(-5-7)^2}=\sqrt{36+144}=6\sqrt{5}\text{ units}$$

$$MN+NS=3\sqrt{5}+3\sqrt{5}=6\sqrt{5}=MS$$

$\because MN+NS=MS$

$\therefore$ M, N and S are collinear points.

Answer 16.



Let O (x, y) be the centre of the circle.
OA = OB (radii of the same circle)

$\Rightarrow OA^2 = OB^2$

$(x-8)^2 + (y-12)^2 = (x-11)^2 + (y-3)^2$

$\Rightarrow x^2 + 64 - 16x + y^2 + 144 - 24y = x^2 + 121 - 22x + y^2 + 9 - 6y$

$\Rightarrow 6x - 18y + 78 = 0$

$\Rightarrow x - 3y + 13 = 0 \quad .....(1)$

similarly, $OB = OC$

$\therefore OB^2 = OC^2$

$(x-11)^2 + (y-3)^2 = (x-0)^2 + (y-14)^2$

$\Rightarrow x^2 + 121 - 22x + y^2 + 9 - 6y = x^2 + y^2 + 196 - 28y$

$\Rightarrow -22x + 22y - 66 = 0$

$\Rightarrow -x + y - 3 = 0 \quad .....(2)$

$x - 3y + 13 = 0 \quad .....(1)$

solving (1) & (2) we get,

$-2y + 10 = 0$

$\Rightarrow y = 5$

from (1)

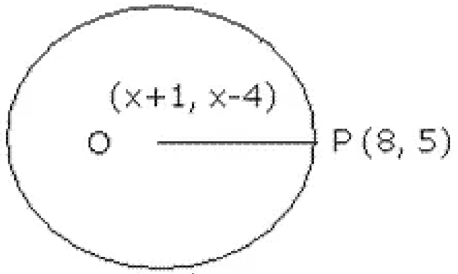
$x - 15 + 13 = 0$

$\Rightarrow x = 2$

Thus, coordinates of O are (2,5)

$\text{Radius} = \sqrt{(2-8)^2 + (5-12)^2} = \sqrt{36+49} = \sqrt{85}\text{units}$

Answer 19.



Given diameter of the circle = 20 units.

$\therefore$ radius = 10units

$OP = 10$

$\sqrt{(x+1-8)^2+(x-4-5)^2} = 10$

squaring both sides,

$x^2+49-14x+x^2 = 81-18x = 100$

$\Rightarrow 2x^2-32x+30=0$

$\Rightarrow x^2-16x+15=0$

$\Rightarrow x^2-15x-x+15=0$

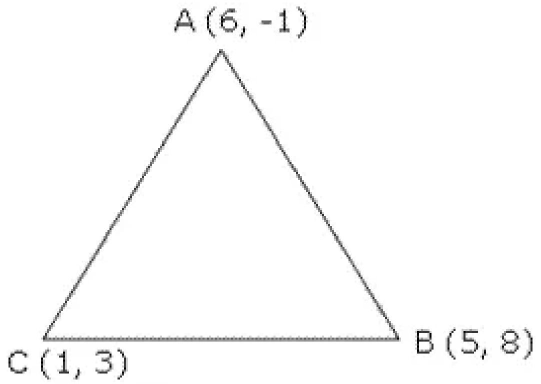
$\Rightarrow (x-15)\ (x-1)=0$

$\Rightarrow x=15$ or 1

Coordinates of O when $x=15$ are $(16, 11)$

Coordinates of O when $x=1$ are $(2,-3)$

Answer 23.



$AB = \sqrt{(6-5)^2+(-1-8)^2} = \sqrt{1+81} = \sqrt{82}$ units

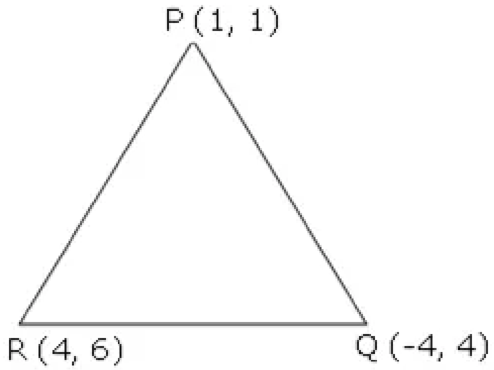
$BC = \sqrt{(5-1)^2+(8-3)^2} = \sqrt{16+25} = \sqrt{41}$ units

$AC = \sqrt{(1-6)^2+(3-1)^2} = \sqrt{25+16} = \sqrt{41}$ units

$\because BC = AC$,

$\therefore$ A, B and C are the vertices of an isosceles triangle.

Answer 24.



$$PQ = \sqrt{(1+4)^2 + (1-4)^2} = \sqrt{25+9} = \sqrt{34} \text{ units}$$

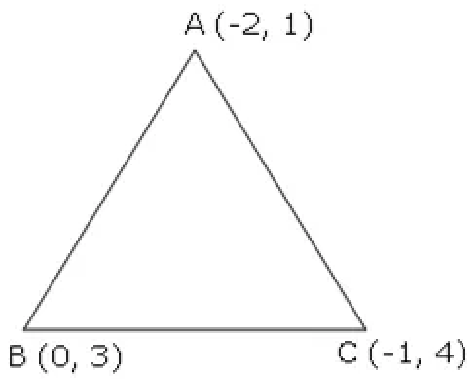
$$QR = \sqrt{(-4-4)^2 + (4-6)^2} = \sqrt{64+4} = \sqrt{68} \text{ units}$$

$$PR = \sqrt{(4-1)^2 + (6-1)^2} = \sqrt{9+25} = \sqrt{34} \text{ units}$$

$\because$ PQ = QR,

$\therefore$ P, Q and R are the vertices of an isosceles triangle

Answer 25.



$$AB = \sqrt{(-2-0)^2 + (1-3)^2} = \sqrt{4+4} = \sqrt{8} \text{ units}$$

$$BC = \sqrt{(10+1)^2 + (3-4)^2} = \sqrt{1+1} = \sqrt{2} \text{ units}$$

$$AC = \sqrt{(-2+1)^2 + (1-4)^2} = \sqrt{1+9} = \sqrt{10} \text{ units}$$

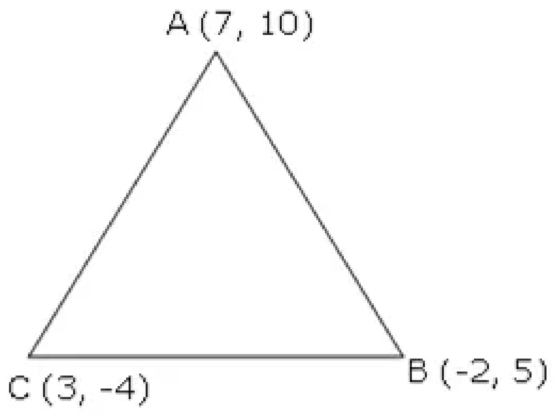
$$AB^2 + BC^2 = 8 + 2 = 10$$

$$AC^2 = 10$$

$\because AB^2 + BC^2 = AC^2$

$\therefore$ A, B and C are the vertices of a right angled triangle.

Answer 26.



$AB = \sqrt{(7+2)^2 + (10-5)^2} = \sqrt{81+25} = \sqrt{106}$ units

$BC = \sqrt{(-2-3)^2 + (5+4)^2} = \sqrt{25+81} = \sqrt{106}$ units

$AC = \sqrt{(7-3)^2 + (10+4)^2} = \sqrt{16+196} = \sqrt{212}$ units

$\because$ $AB = BC$

$\therefore$ ABC is an isosceles triangle.

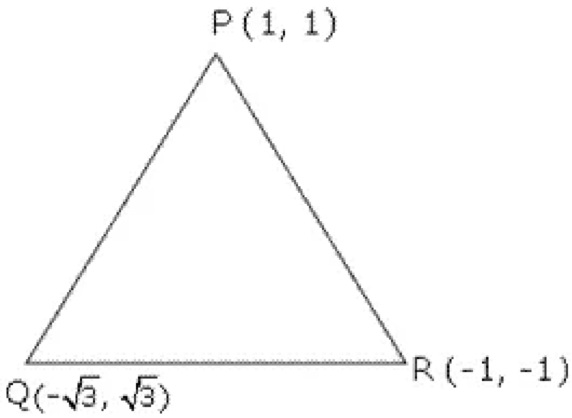
$AB^2 + BC^2 = 100 + 106 = 212$

$AC^2 = 212$

$\because AB^2 + BC^2 = AC^2$

$\therefore$ ABC is also a right angled triangle.

Answer 27.



$$PQ = \sqrt{\left(1+\sqrt{3}\right)^2 + \left(1-\sqrt{3}\right)^2} = \sqrt{4+4} = \sqrt{8}\text{units}$$

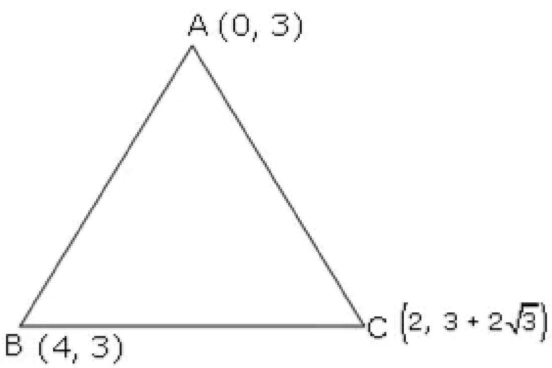
$$QR = \sqrt{\left(-\sqrt{3}+1\right)^2 + \left(\sqrt{3}+1\right)^2} = \sqrt{4+4} = \sqrt{8}\text{units}$$

$$PR = \sqrt{(-1-1)^2 + (-1-1)^2} = \sqrt{4+4} = \sqrt{8}\text{units}$$

$\because\ PQ = QR = PR$

$\therefore$ PQR is an equilateral triangle

Answer 28.



$$AB = \sqrt{(0-4)^2 + (3-3)^2} = 4\text{units}$$

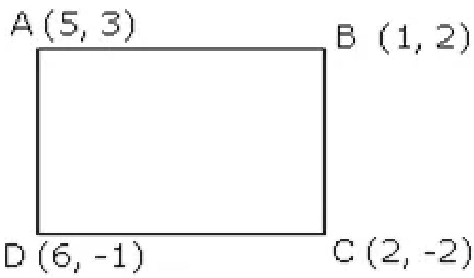
$$BC = \sqrt{(4-2)^2 + \left(3-3-2\sqrt{3}\right)^2} = \sqrt{4+12} = 4\text{units}$$

$$AC = \sqrt{(2-0)^2 + \left(3+2\sqrt{3}-3\right)^2} = \sqrt{4+12} = 4\text{units}$$

$\because\ AB = BC = AC$

$\therefore$ ABCisan equilateral triangle.

Answer 29.



$$AB=\sqrt{(5-1)^2+(3-2)^2}=\sqrt{16+1}=\sqrt{17}\text{ units}$$

$$BC=\sqrt{(1-2)^2+(2+2)^2}=\sqrt{1+16}=\sqrt{17}\text{ units}$$

$$CD=\sqrt{(6-2)^2+(-1+2)^2}=\sqrt{16+1}=\sqrt{17}\text{ units}$$

$$DA=\sqrt{(6-5)^2+(-1-3)^2}=\sqrt{1+16}=\sqrt{17}\text{ units}$$

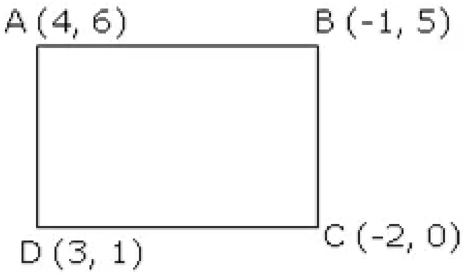
$$AC=\sqrt{(5-2)^2+(3+2)^2}=\sqrt{9+25}=\sqrt{34}\text{ units}$$

$$BD=\sqrt{(6-1)^2+(-1-2)^2}=\sqrt{25+9}=\sqrt{34}\text{ units}$$

$\because AB=BC=CD=DA$ and $AC=BD$

$\therefore$ ABCD is asquare.

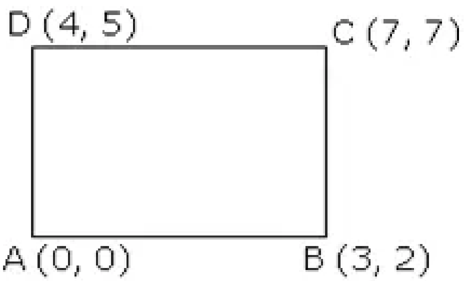
Answer 30.



$$AB = \sqrt{(4+1)^2 + (6-5)^2} = \sqrt{25+1} = \sqrt{26}\,\text{units}$$
$$BC = \sqrt{(-1+2)^2 + (5-0)^2} = \sqrt{1+25} = \sqrt{26}\,\text{units}$$
$$CD = \sqrt{(-2-3)^2 + (0-1)^2} = \sqrt{25+1} = \sqrt{26}\,\text{units}$$
$$DA = \sqrt{(3-4)^2 + (1-6)^2} = \sqrt{1+25} = \sqrt{26}\,\text{units}$$
$$AC = \sqrt{(4+2)^2 + (6-0)^2} = \sqrt{36+36} = 36\sqrt{2}\,\text{units}$$
$$BD = \sqrt{(-1-3)^2 + (5-1)^2} = \sqrt{36+36} = 16\sqrt{2}\,\text{units}$$
$\because AB = BC = CD = DA$ and $AC \neq BD$

$\therefore$ ABCD is a rhombus

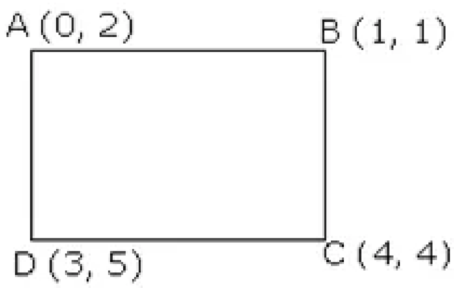
Answer 31.



$$AB = \sqrt{(3-0)^2 + (2-0)^2} = \sqrt{9+4} = \sqrt{13}\,\text{units}$$
$$BC = \sqrt{(3-7)^2 + (2-7)^2} = \sqrt{16+25} = \sqrt{41}\,\text{units}$$
$$CD = \sqrt{(7-4)^2 + (7-5)^2} = \sqrt{9+4} = \sqrt{13}\,\text{units}$$
$$DA = \sqrt{(4-0)^2 + (5-0)^2} = \sqrt{16+25} = \sqrt{41}\,\text{units}$$
$\because AB = CD$ and $BC = DA$

$\therefore$ ABCD is a parallelogram.

Answer 32.



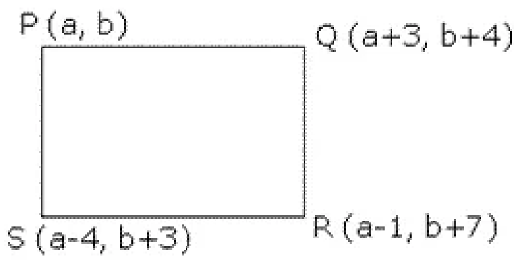
$$AB=\sqrt{(0-1)^2+(2-1)^2}=\sqrt{2}\text{units}$$
$$BC=\sqrt{(1-4)^2+(1-4)^2}=3\sqrt{2}\text{units}$$
$$CD=\sqrt{(4-3)^2+(4-5)^2}=\sqrt{2}\text{units}$$
$$DA=\sqrt{(3-0)^2+(5-2)^2}=3\sqrt{2}\text{units}$$
$$AC=\sqrt{(4-0)^2+(4-2)^2}=\sqrt{20}=2\sqrt{5}\text{units}$$
$$BC=\sqrt{(3-1)^2+(5-1)^2}=\sqrt{20}=2\sqrt{5}\text{units}$$

$\because AB=CD$ and $BC=DA$

Also, $AC=BD$

$\therefore$ ABCD is a rectangle.

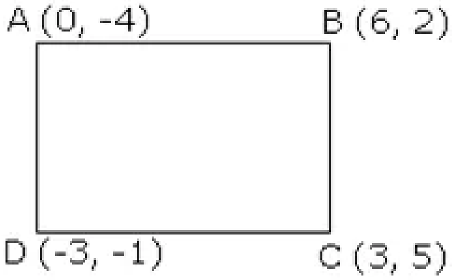
Answer 33.



$$PQ=\sqrt{(a+3-a)^2+(b+4-b)^2}=\sqrt{9+16}=5\text{units}$$
$$QR=\sqrt{(a+3-a+1)^2+(b-4-b-7)^2}=\sqrt{16+9}=5\text{units}$$
$$RS=\sqrt{(a-1-a+4)^2+(b+7-b-3)^2}=\sqrt{9+16}=5\text{units}$$
$$SP=\sqrt{(a-4-a)^2+(b+3-b)^2}=\sqrt{16+9}=5\text{units}$$

Since the opposite sides of quadilateral PQRS are equal, therefore, it is a parallelogram.

Answer 34.



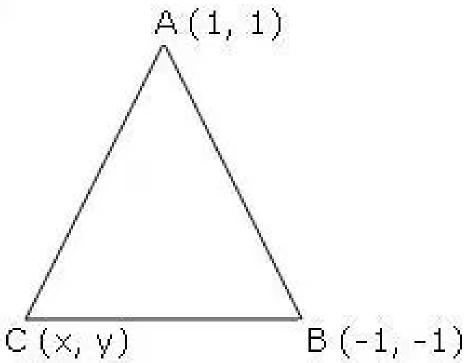
$$AB=\sqrt{(6-0)^2+(2+4)^2}=6\sqrt{2}\text{ units}$$
$$BC=\sqrt{(6-3)^2+(2-5)^2}=3\sqrt{2}\text{ units}$$
$$CD=\sqrt{(3+3)^2+(5+1)^2}=6\sqrt{2}\text{ units}$$
$$DA=\sqrt{(-3-0)^2+(-1+4)^2}=3\sqrt{2}\text{ units}$$
$$AC=\sqrt{(3-0)^2+(5+4)^2}=3\sqrt{10}\text{ units}$$
$$BD=\sqrt{(6+3)^2+(2+1)^2}=3\sqrt{10}\text{ units}$$

$\because AB=CD$ and $BC=DA$,

Also $AC=BD$

$\therefore ABCD$ is a rectangle.

Answer 37.



ABC is an equilateral triangle.

$\therefore AC = BC \quad \text{and} \quad AB = BC$

$\Rightarrow AC^2 = BC^2 \quad \text{and} \quad AB^2 = BC^2$

$(x-1)^2 + (y-1)^2 = (x+1)^2 + (y+1)^2$

$\Rightarrow x^2 + 1 - 2x + y^2 + 1 - 2y = x^2 + 1 + 2x + y^2 + 1 + 2y$

$\Rightarrow -4x - 4y = 0$

$\Rightarrow -4x = 4y$

$\Rightarrow x = -y \quad \ldots\ldots..(1)$

$(1+1)^2 + (1+1)^2 = (x+1)^2 + (y+1)^2$

$\Rightarrow 8 = x^2 + 1 + 2x + y^2 + 1 + 2y$

$\Rightarrow 8 = y^2 + 1 - 2y + y^2 + 1 + 2y$

$\Rightarrow 2y^2 - 6 = 0$

$\Rightarrow y^2 = 3$

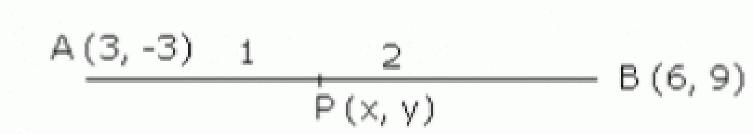
$\Rightarrow y = \pm\sqrt{3}$

from(1)

$\therefore x = \mp\sqrt{3}$

Ex 12.2

Answer 1.



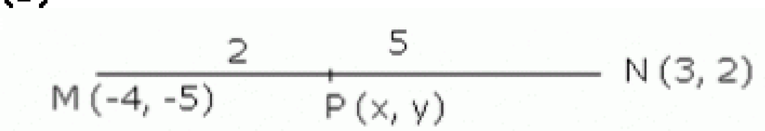
Let the point P divides the line segment AB in the ratio 1:2.

$\therefore$ coordinates of P are

$$x=\frac{1\times 6+2\times 3}{1+2}=4$$

$$y=\frac{1\times 9+2\times -3}{1+2}=1$$

(b)



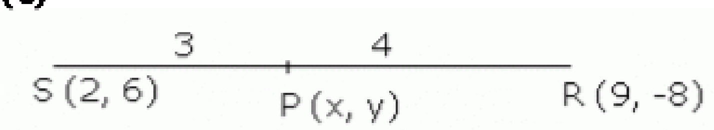
Let the point P divides the line segment MN in the ratio 2:5.

$\therefore$ coordinates of P are

$$x=\frac{2\times 3+5\times -4}{2+5}=\frac{-14}{7}=-2$$

$$y=\frac{2\times 2+5\times -5}{2+5}=-3$$

(c)



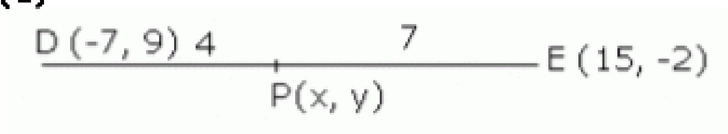
Let the point P divides the line segment SR in the ratio 3:4.

$\therefore$ coordinates of P are

$$x=\frac{3\times 9+4\times 2}{3+4}=5$$

$$y=\frac{3\times -8+4\times 6}{3+4}=0$$

(d)



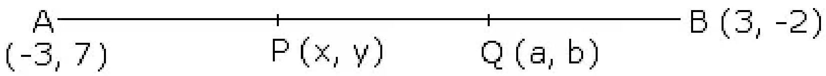
Let the point P divides DE in the ratio 4:7.

$\therefore$ coordinates of P are

$$x=\frac{4\times 15+7\times -7}{4+7}=1$$

$$y=\frac{4\times -2+7\times 9}{4+7}=5$$

Answer 2.



Let P (x, y) and Q (a, b) be the point of trisection of the line segment AB.

AP: PB = 1:2

Coordinates of P are

$$x=\frac{1\times3+2\times-3}{1+2}=-1$$

$$y=\frac{1\times-2+2\times7}{1+2}=4$$

$P(-1,4)$

$AQ:QB=2:1$

coordinates of Q are,

$$a=\frac{2\times3=1\times-3}{2+1}=1$$

$$b=\frac{2\times-2+1\times7}{2+1}=1$$

$Q(1,1)$

$\therefore$ The points of trisection are $(-1,4)$ and $(1,1)$.

Answer 3.

Let A (x, y) and B (a, b) be the points of trisection of line segment MN.

MA : AN = 1 : 2

$\therefore$ coordinates of A are,

$$x=\frac{1\times6+2\times3}{1+2}=4$$

$$y=\frac{1\times9+2\times-3}{1+2}=1$$

$A(4,1)$

Also, $MB:BN=2:1$

coordinates of B are,

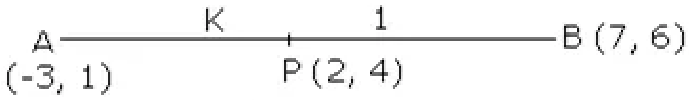
$$a=\frac{2\times6+1\times3}{2+1}=5$$

$$b=\frac{2\times9+1\times-3}{2+1}=5$$

$B(5,5)$

points of trisection are $(4,1)$ and $(5,5)$.

Answer 4.



Let the point P divides AB in the ratio k:1.
Coordinates of P are,

$$x = \frac{7k-3}{k+1}$$

$$y = \frac{6k+1}{K+1}$$

But given, $P(x, y) = P(2, 4)$

$$\therefore 2 = \frac{7k-3}{k+1}$$

$$\Rightarrow 2k+2 = 7k-3$$

$$\Rightarrow 5 = 5k$$

$$\Rightarrow k = 1$$

$$k : 1 = 1 : 1$$

$$\text{or} \quad 4 = \frac{6k+1}{k+1}$$

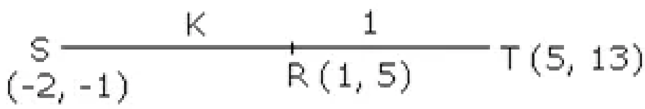
$$4k+4 = 6k+1$$

$$\Rightarrow 3 = 2k$$

$$\Rightarrow k = \frac{3}{2}$$

$$k : 1 = 3 : 2$$

Answer 5.



Let R divides the line segment ST in the ratio k: 1.
Coordinates of R
R (x, y) = R (1, 5)

$$R\left[\left(\frac{5k-2}{k+1}, \frac{13k-1}{k+1}\right)\right] = R(1, 5)$$

$$\frac{5k-2}{k+1} = 1$$

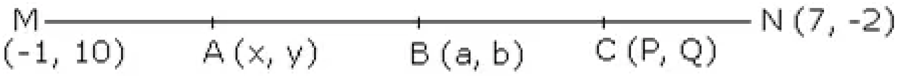
$$5k-2 = k+1$$

$$4k = 3$$

$$k = \frac{3}{4}$$

Hence, required ratio is $k : 1 = 3 : 4$.

Answer 6.



Given, A (x, y), B (a, b) and C (p, q) divides the line segment MN in four equal parts. B in the mid point of MN. i.e. MB :BN = 1:1
Coordinates of B are,

$$B(a,b)=B\left(\frac{7-1}{2},\frac{-2+10}{2}\right)=B(3,4)$$

A is the mid point of MB i.e. MA : AB = 1 : 1
coordinates of A are.

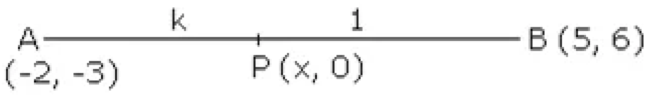
$$A(x,y)=A\left(\frac{3-1}{2},\frac{4+10}{2}\right)=A(1,7)$$

C is the mid point of BN i.e BC : CN = 1 : 1

$$C(p,q)=C\left(\frac{3+7}{2},\frac{4-2}{2}\right)=C(5,1)$$

Hence, the coordinates of A, B and C are $(1,7),(3,4)$ and $(5,1)$ respectively.

Answer 7.



Let the point on x-axis be P (x, 0) which divides the line segment AB in the ratio k: 1.
Coordinates of P are

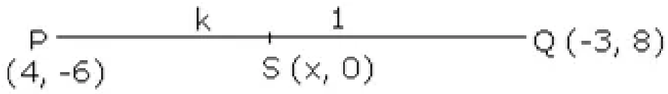
$$x=\frac{5k+2}{k+1},\quad 0=\frac{6k-3}{K+1}$$

$$\Rightarrow 0=6k-3$$

$$k=\frac{1}{2}$$

Hence, the required ratio is 1 : 2.

Answer 8.



Given PQ is divided by the line Y = 0 i.e. x-axis.
Let S (x, 0) be the point on line Y = 0, which divides the line segment PQ in the ratio k: 1.
Coordinates of S are

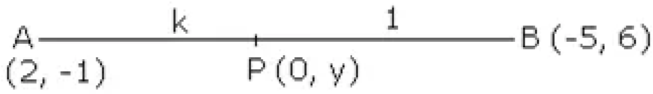
$$x=\frac{-3k+4}{k+1},\ 0=\frac{8k-6}{k+1}$$

$$\Rightarrow 8k=6$$

$$\Rightarrow k=\frac{3}{4}$$

Hence, the required ratio is 3 : 4.

Answer 9.



Let the point P (0, y) lies on y-axis which divides the line segment AB in the ratio k: 1.
Coordinates of P are,

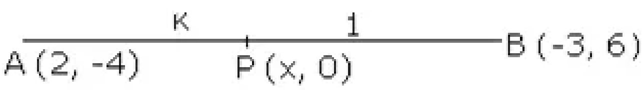
$$0=\frac{-5k+2}{k+1},\ y=\frac{6k-1}{k+1}$$

$$\Rightarrow 5k=2$$

$$\Rightarrow k=\frac{2}{5}$$

Hence, the required ratio is 2 : 5.

Answer 10.



Let P (x, 0) be the point on line y = 0 i.e. x-axis which divides the line segment AB in the ratio k: 1.
Coordinates of P are

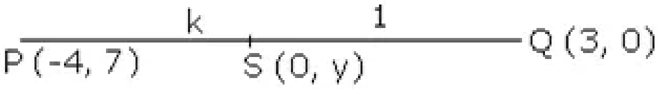
$$x=\frac{-3k+2}{k+1},\ 0=\frac{6k-4}{k+1}$$

$$\Rightarrow 6k=4$$

$$\Rightarrow k=\frac{2}{3}$$

Hence the required ratio is 2 : 3.

Answer 11.



Let S (0, y) be the point on line x = 0 i.e. y-axis which divides the line segment PQ in the ratio k: 1.
Coordinates of S are,

$$0=\frac{3k-4}{k+1},\quad Y=\frac{0+7}{k+1}$$

$$\Rightarrow 3k=4$$

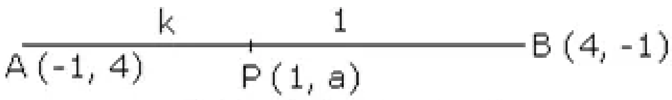
$$k=\frac{4}{3}\quad -(1)$$

$$Y=\frac{7}{\frac{4}{3}+1}\quad (\text{from}(1))$$

$$Y=3$$

Hence, the required ratio is 4 : 3 and the required point is $S(0, 3)$.

Answer 12.



Let the point P (1, a) divides the line segment AB in the ratio k: 1.
Coordinates of P are,

$$1=\frac{4k-1}{k+1},$$

$$\Rightarrow k+1=4k-1$$

$$\Rightarrow 2=3k$$

$$\Rightarrow k=\frac{2}{3}\quad \ldots(1)$$

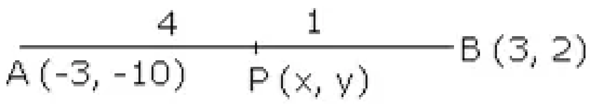
$$\Rightarrow a=\frac{-k+4}{k+1}$$

$$\Rightarrow a=\frac{\frac{-2}{3}+4}{\frac{2}{3}+1}\quad (\text{from}(1))$$

$$\Rightarrow a=\frac{10}{5}=2$$

Hence, the required ratio is 2 : 3 and the value of a is 2.

Answer 13.



Given : $-PB : AB = 1 : 5$

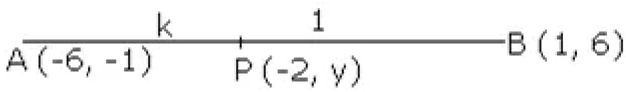
$\therefore PB : PA = 1 : 4$

Coordinates of P are

$$(x, y) = \left(\frac{4 \times 3 - 3}{5}, \frac{4 \times 2 - 10}{5}\right) = \left(\frac{9}{5}, \frac{-2}{5}\right)$$

$$P\left(\frac{9}{5}, \frac{-2}{5}\right)$$

Answer 14.



Let P (-2, y) be the point on line x which divides the line segment AB the ratio k: 1.

Coordinates of P are,

$$-2 = \frac{k-6}{k+1},$$

$$\Rightarrow -2k - 2 = k - 6$$

$$\Rightarrow -3k = -4$$

$$\Rightarrow k = \frac{4}{3} \quad \ldots.(1)$$

$$y = \frac{6k-1}{k+1}$$

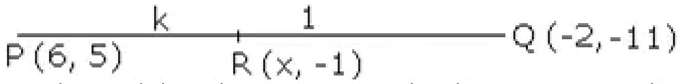
$$\Rightarrow y = \frac{69\left(\frac{4}{3}\right) - 1}{\frac{4}{3} + 1} \quad (\text{from } (1))$$

$$\Rightarrow y = \frac{24-3}{7}$$

$$\Rightarrow y = 3$$

Hence, the required ratio is 4: 3 and the point of intersection is (-2, 3).

Answer 15.



Let R (x, -1) be the point on the line y = -1 which divides the line segment PQ in the ratio k: 1.
Coordinates of R are,

$$x=\frac{-2k+6}{k+1}, \quad -1=\frac{-11k+5}{k+1}$$

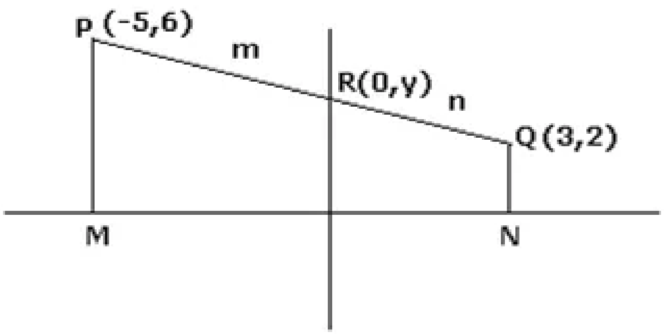
$$x=\frac{-2\left(\frac{3}{5}\right)+6}{\frac{3}{5}+1}, \quad \Rightarrow -k-1=-11k+5$$

$$\Rightarrow x=\frac{-6+30}{8} \quad \Rightarrow 10k=6$$

$$x=3 \quad \Rightarrow k=\frac{3}{5} \quad ....(1)$$

Hence, the required ratio is $3:5$ and the point of intersection is $(3,-1)$.

Answer 16.



R(0,y) is the point on the y-axis that divides PQ.
Let the ratio in which PQ is divided by R be m:n.
Now, R(o,y), $(x_1,y_1)=(-5,6)$ and $(x_2,y_2)=(3,2)$ and the ratio is m:n.

$$0=\frac{mx_2+nx_1}{m+n}$$

$$\Rightarrow 0=\frac{3m-5n}{m+n}$$

$$\Rightarrow 0=3m-5n$$

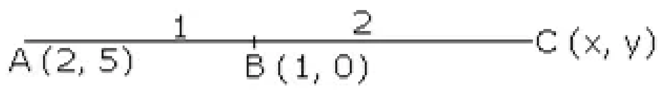
$$\Rightarrow 3m=5n$$

$$\Rightarrow \frac{m}{n}=\frac{5}{3}$$

$$\Rightarrow m:n=5:3$$

$$\Rightarrow PR:RQ=5:3$$

Answer 17.



Given $AC : AB = 3 : 1$

$\therefore AB : BC = 1 : 2$

Coordinates of B are

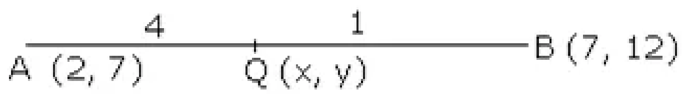
$1 = \frac{x+4}{3}, \quad 0 = \frac{y+10}{3}$

$3 = x+4, \quad 0 = y+10$

$x = -1, \quad y = -10$

Hence the coordinates of C are $(-1, -10)$.

Answer 18.



$AQ : BQ = 4 : 1$

Coordinates of Q are

$Q(x, y) = Q\left(\frac{4\times7+1\times2}{4+1}, \frac{4\times12+1\times7}{4+1}\right) = Q(6, 11)$

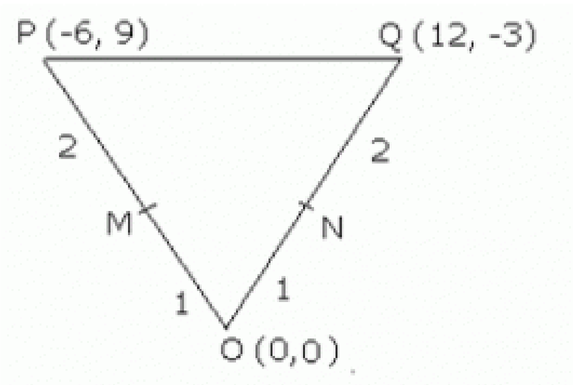
Thus the coordinates of Q are $(6, 11)$.

$AQ = \sqrt{(2-6)^2+(7-11)^2} = \sqrt{16+16} = 4\sqrt{2}$

$BQ = \sqrt{(7-6)^2+(12-11)^2} = \sqrt{1+1} = \sqrt{2}$

$\Rightarrow AQ = 4BQ$

Answer 19.



It is given that M divides OP in the ratio 1: 2 and point N divides OQ in the ratio 1: 2.

Using section formula, the coordinates of M are

$$\left(\frac{-6+0}{3}, \frac{9+0}{3}\right) = (-2, 3)$$

Using section formula, the coordinates of N are

$$\left(\frac{12+0}{3}, \frac{-3+0}{3}\right) = (4, -1)$$

Thus, the coordinates of M and N are (-2, 3) and (4, -1) respectively.

Now, using distance formula, we have:

$$PQ = \sqrt{(-6-12)^2 + (9+3)^2} = \sqrt{324 + 144} = \sqrt{468}$$

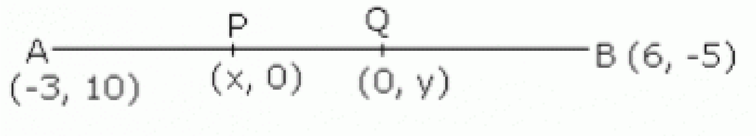
$$MN = \sqrt{(4+2)^2 + (-1-3)^2} = \sqrt{36 + 36} = \sqrt{52}$$

It can be observed that:

$$PQ = \sqrt{468} = \sqrt{9 \times 52} = 3\sqrt{52} = 3MN$$

Hence, proved.

Answer 22.



Let the coordinates of two points x-axis and y-axis be P (x, 0) and Q (0, y) respectively. Let P divides AB in the ratio k: 1.

Coordinates of P are

$$P(x, o)=P\left(\frac{6k-3}{k+1}, \frac{-5k+10}{k+1}\right)$$

$$\Rightarrow 0=\frac{-5k+10}{k+1}$$

$$\Rightarrow 5k=10$$

$$\Rightarrow k=2$$

Hence P divides AB in the ratio 2 : 1.

Let Q divides AB in the ratio k_1 : 1.

Coordinates of Q are,

$$Q(0, y)=Q\left(\frac{6k_1-3}{k+1}, \frac{-5k_1+10}{k+1}\right)$$

$$\Rightarrow 0=\frac{6k_1-3}{k=1}$$

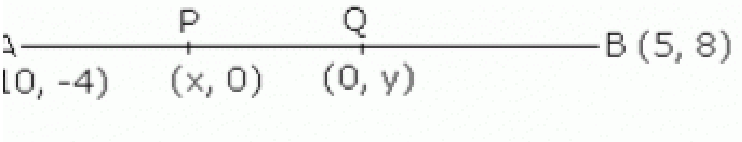
$$\Rightarrow 6k_1=3$$

$$\Rightarrow k_1=\frac{1}{2}$$

Hence Q divides AB in the ratio 1 : 2

Hence proved, P and Q are the points of trisection.

Answer 23.



Let P (x, 0) lies on the line y = 0 i.e. x-axis and divides the line segment AB in the ratio k: 1.
Coordinates of P are,

$$P(x,0)=P\left(\frac{5k-10}{k+1},\frac{8k-4}{k+1}\right)$$

$$\Rightarrow 0=\frac{8k-4}{k+1}, \qquad \frac{5k-10}{k+1}=x$$

$$\Rightarrow 8k=4, \qquad \frac{5\left(\frac{1}{2}\right)-10}{\frac{1}{2}+x}=x \qquad \text{(from(1))}$$

$$\Rightarrow k=\frac{1}{2} \quad ....(1), \qquad x=-5$$

Hence $P(-5,0)$ divides AB in the ratio 1 : 2.

Let Q (0, y) lies on the line x = 0 i.e. y – axis and divides the line segment AB in the ratio $k_1 : 1$.

Coordinates of Q are

$$Q(0,y)=Q\left(\frac{5k_1-10}{k_1+1},\frac{8k_1-4}{k_1+1}\right)$$

$$0=\frac{5k_1-10}{k_1+1}, \qquad y=\frac{8k_1-4}{k_1+1}$$

$$\Rightarrow 5k_1=10, \qquad y=\frac{8(2)-4}{2+1} \quad \text{(from(2))}$$

$$\Rightarrow k_1=2 \quad ....(2) \quad y=4$$

Hence, $Q(0,4)$ divides in the ratio 2 : 1.

Hence proved P and Q are the points of trisection of AB.

Ex 12.3

Answer 1.

(a)

A(4,7) P(x,y) B(10,15)

Coordinates of P are

$$P(x,y) = P\left(\frac{4+10}{2}, \frac{7+15}{2}\right)$$

$$= P(7,11)$$

(b)

P(-3,5) R(x,y) Q(9,-9)

Coordinates of R are,

$$R(x,y) = R\left(\frac{-3+9}{2}, \frac{5-9}{2}\right)$$

$$= R(3,-2)$$

(c)

M(a+b,b-a) O(x,y) N(a-b,a+b)

Coordinates of O are,

$$O(x,y) = O\left(\frac{a+\cancel{b}+a-\cancel{b}}{2}, \frac{b-\cancel{a}+\cancel{a}+b}{2}\right)$$

$$= O(a,b)$$

(d)

A(3a-2b,5a+7b) C(X,Y) B(a+4b,a-3b)

Coordinates of C are,

$$C(x,y) = C\left(\frac{a+4b+3a-2b}{2}, \frac{a-3b+5a+7b}{2}\right)$$

$$= C(2a+b, 3a+2b)$$

(e)

P(a+3,5b) R(x,y) Q(3a-1,3b+4)

Coordinates of R are,

$$R(x,y) = R\left(\frac{a+3+3a-1}{2}, \frac{5b+3b+4}{2}\right)$$

$$= R(2a+1, 4b+2)$$

Answer 2.

Coordinates of P are,

$$P(6,3) = P\left(\frac{-2+x}{2}, \frac{0+y}{2}\right)$$

$$6 = \frac{-2+x}{2}, \qquad 3 = \frac{y}{2}$$

$\Rightarrow$ 12 = -2 + x , $\qquad$ y = 6

$\Rightarrow$ x = 14

Coordinates of B are(14,6).

Answer 3.

Coordinates of B are (14,6)

Let A(x,0) lies on x-axis and B(0,y) lies on y-axis, given AP : PB = 1 : 1

Coordinates of P are,

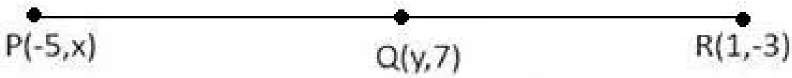
$$P(4,-3) = P\left(\frac{x+0}{2}, \frac{0+y}{2}\right)$$

$$4 = \frac{x}{2}, \quad -3 = \frac{4}{2}$$

x = 8 , y = -6

Co-ordinates of A are (8,0) and B are (0,-6)

Answer 4.



Given PQ = PR, i.e. PQ : QR = 1 : 1

Coordinates of Q are,

$$Q(y,7) = Q\left(\frac{1-5}{2}, \frac{-3+x}{2}\right)$$

$$y = -2, \; 7 = \frac{-3+x}{2}$$

y = -2, 14= -3+x

17 = x

The values of x and y are 17 and -2 respectively.

Answer 5.

$$\frac{AB}{AC} = \frac{1}{2}$$

$\therefore$ AB : BC = 1 : 1

Coordinates of B are,

$$B(-2,b) = B\left(\frac{-4+a}{2}, \frac{-4+2}{2}\right)$$

$$-2 = \frac{-4+a}{2}, \quad b = -1$$

$$-4 = -4 + a \quad , \quad b = -1$$

The values of a and b are 0 and -1 respectively

Answer 6.

Given : PR : RQ = 1 : 1

Coordinates of R are,

$$R(3,5) = R\left(\frac{2+n}{2}, \frac{m+4}{2}\right)$$

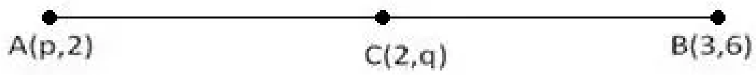
$$B = \frac{2+n}{2}, \quad 5 = \frac{m+4}{2}$$

$$6 = 2 + n \quad , \quad 10 = m + 4$$

n = 4, m= 6

The values of m and n are 6 and 4 respectively.

Answer 7.



AC: CB = 1 : 1

Coordinates of C are,

$$C(2,q) = C\left(\frac{p+3}{2}, \frac{2+6}{2}\right)$$

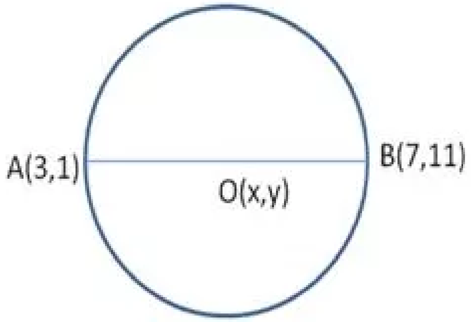
$$2 = \frac{p+3}{2}, q = 4$$

$$4 = p + 3, q=4$$

$$p = 1, q=4$$

the values of p and q are 1 and 4 respectively.

Answer 8.



Let O(x,y) be the centre of the circle with diameter AB,

$\therefore$ O is midpoint of AB

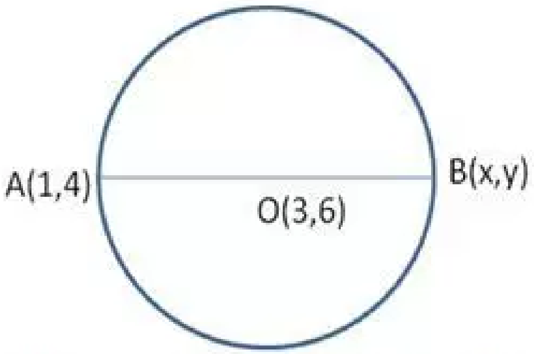
i.e. AO : OB = 1 : 1

Coordinates of O are,

$$O(x,y) = O\left(\frac{3+7}{2}, \frac{1+11}{2}\right) = O(5,6)$$

Thus, the coordinates of centre are (5,6).

Answer 9.



O is the centre of the circle with diameter AB.

$\therefore$ AO : OB = 1 : 1

Coordinates of O are,

$$O(3,6) = O\left(\frac{1+x}{2}, \frac{4+y}{2}\right)$$

$$3 = \frac{1+x}{2}, \quad 6 = \frac{4+y}{2}$$

$6 = 1 + x$, $\quad 12 = 4 + y$

$x = 5$, $y = 8$

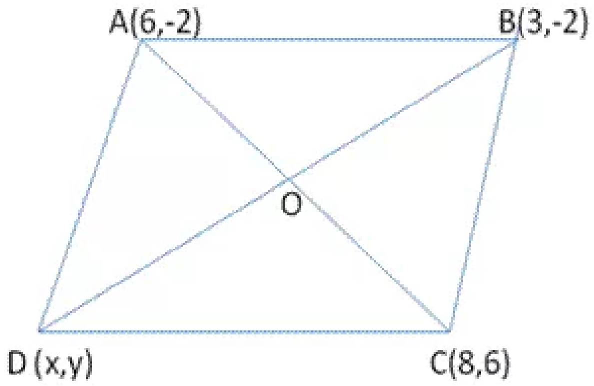
Coordinates of B are (5,8)

$$\text{Length of AB} = \sqrt{(5-1)^2 + (8-4)^2}$$

$$= \sqrt{16 + 16}$$

$$= 4\sqrt{2}\text{units}$$

Answer 10.



We know that in a parallelogram, diagonals bisect each other.

$\therefore$ midpoint of AC = midpoint of BD

$$O\left(\frac{6+8}{2},\frac{-2+6}{2}\right)=O\left(\frac{x+3}{2},\frac{y-2}{2}\right)$$

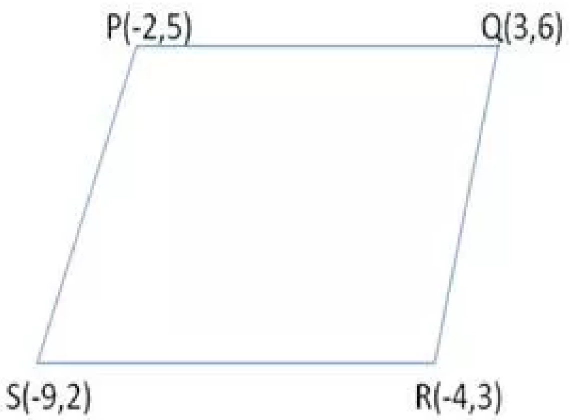
$$\therefore \frac{6+8}{2}=\frac{x+3}{2},\frac{-2+6}{2}=\frac{y-2}{2}$$

$14 = x + 3,\quad 4 = y - 2$

$x = 11,\ y = 6$

the coordinates of the fourth vertex D are (11,6)

Answer 11.



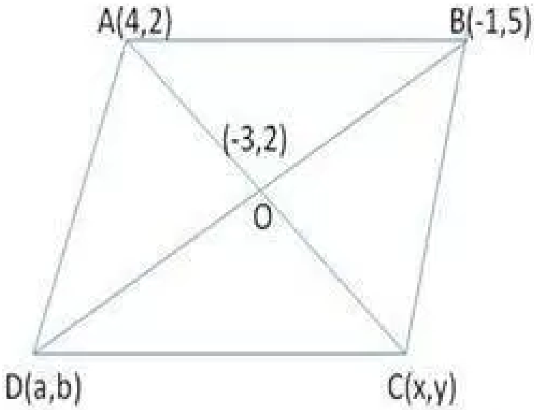
Coordinates of midpoint of PR are $\left(\frac{-2-4}{2},\frac{5+3}{2}\right)$ i.e. (-3,4)

Coordinates of midpoint of QS are $\left(\frac{-9+3}{2},\frac{2+6}{2}\right)$ i.e. (-3,4)

The midpoint of PR is same as that of QS, i.e. diagonals PR and QS bisect each other.

Hence, PQRS is a parallelogram.

Answer 12.



Let the coordinates of C and D be (x,y) and (a,b) respectively
Midpoint of AC is O coordinates of O are,

$$O(-3,2) = O\left(\frac{4+x}{2}, \frac{2+y}{2}\right)$$

$$-3 = \frac{4+x}{2}, 2 = \frac{2+y}{2}$$

$-6 = 4 + x, \quad 4 = 2 + y$

$x = -10 \;, \; y = 2$

C(-10,2)

Similarly, coordinates of midpoint of DB, i.e. O are,

$$O(-3,2) = O\left(\frac{a-1}{2}, \frac{b+5}{2}\right)$$

$$-3 = \frac{a-1}{2}, 2 = \frac{b+5}{2}$$

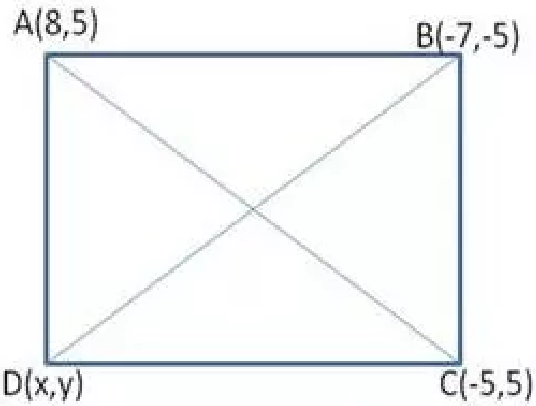
$-6 = a - 1, 4 = b + 5$

$a = -5, b = -1$

$D(-5,-1)$

Thus, the coordinates of the other two vertices are (-10,2) and (-5,-1)

Answer 13.



we know that in a parallelogram diagonals bisect each other

$\therefore$ midpoint of AC = midpoint of BD

$$O\left(\frac{8-5}{2},\frac{5+5}{2}\right)=O\left(\frac{x-7}{2},\frac{y-5}{2}\right)$$

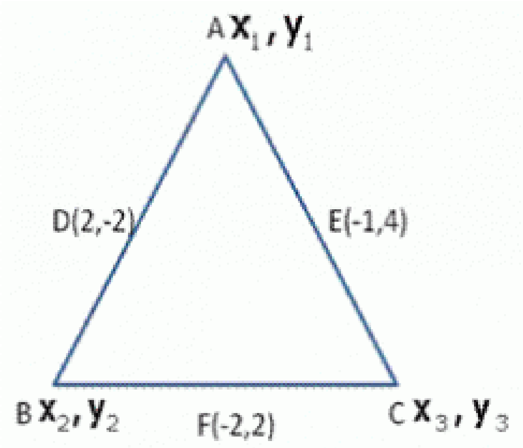
$$\frac{8-5}{2}=\frac{x-7}{2},\frac{5+5}{2}=\frac{y-5}{2}$$

$$\frac{3}{2}=\frac{x-7}{2},10=y-5$$

$x = 10, y = 15$

Coordinates of fourth vertex D are (10,15)

Answer 14.



Let $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ be the coordinates of the vertices ΔABC.

Midpoint of AB, i.e. D

$$D(2,1) = D\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$2 = \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} = -1$$

$x_1 + x_2 = 4$ --(1) $\quad y_1 + y_2 = -2$ --(2)

Similarly,

$x_1 + x_3 = -2$ --(3) $\quad y_1 + y_3 = 8$ --(4)

$x_2 + x_3 = -4$ --(5) $\quad y_2 + y_3 = 4$ --(6)

Adding (1), (3) and (5)

$2(x_1 + x_2 + x_3) = -2$

$x_1 + x_2 + x_3 = -1$

$4 + x_3 = -1$ [from (1)]

$x_3 = -5$

Adding (2),(4) and (6)

$2(y_1 + y_2 + y_3) = 10$

$y_1 + y_2 + y_3 = 5$

$-2 + y_3 = 5$ [from (2)]

$y_3 = 7$

From (3)

$x_1 - 5 = -2$

$x_1 = 3$

From (4)

$y_1 + 7 = 8$

$y_1 = 1$

From (5)

$x_2 - 5 = -4$

$x_2 = 1$

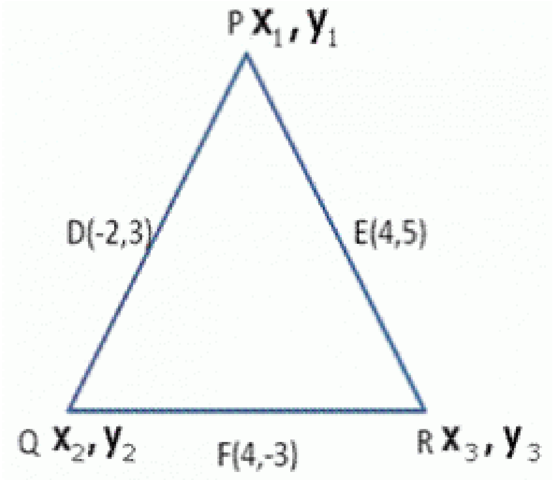
From (6)

$y_2 + 7 = 4$

$y_3 = -3$

The coordinates of the vertices of ΔABC are (3,1), (1,-3) and (-5,7)

Answer 15.



Let $P(x_1, y_1), Q(x_2, y_2)$ and $R(x_3, y_3)$ be the coordinates of the verti

of ΔPQR.

Midpoint of PQ is D

$$D(-2,3) = D\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$\frac{x_1+x_2}{2} = -2, \frac{y_1+y_2}{2} = 3$$

$$x_1 + x_2 = -4 - -(1), y_1 + y_2 = 6 - -(2)$$

similarly,

$$x_2 + x_3 = 8 - -(3), y_2 + y_3 = -6 - -(4)$$

$$x_1 + x_3 = 8 - -(5), y_1 + y_3 = 10 - -(6)$$

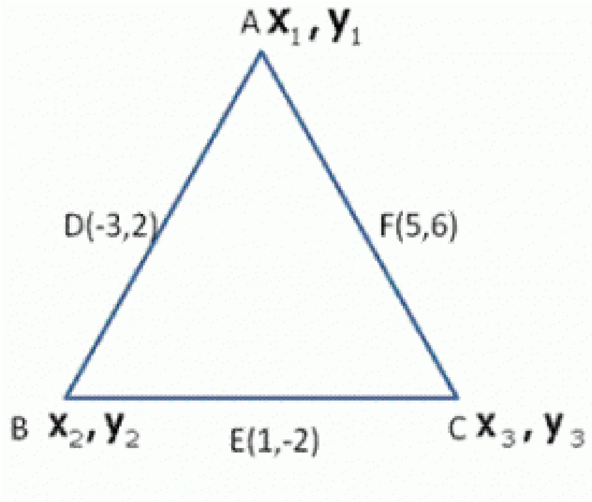
Adding (1), (3) and (5)

$$2(x_1 + x_2 + x_3) = 12$$
$$x_1 + x_2 + x_3 = 6$$
$$-4 + x_3 = 6$$
$$x_3 = 10$$

Adding (2), (4) and (6)

$$2(y_1 + y_2 + y_3) = 10$$
$$y_1 + y_2 + y_3 = 5$$
$$6 + y_3 = 5$$
$$y_3 = -1$$

Answer 16.



let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the coordinates of the vertices of ΔABC.

D is the midpoint of AB<

$$D(-3,2) = D\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$\frac{x_1+x_2}{2} = -3, \frac{y_1+y_2}{2}$$

$x_1 + x_2 = -6 - - - (1)$ $\qquad$ $y_1 + y_2 = 4 - - - (2)$

Similarly

$x_2 + x_3 = 2 - - - (3)$ $\qquad$ $y_2 + y_3 = -4 - - - (4)$

$x_1 + x_3 = 10 - - - (5)$ $\qquad$ $y_1 + y_3 = 12 - - - (6)$

Adding (1), (3) and (5)

$2(x_1 + x_2 + x_3) = 6$

$x_1 + x_2 + x_3 = 3$

$-6 + x_3 = 3$

$x_3 = 9$

From (3)

$x_2 + 9 = 2$

$x_2 = -7$

From (5)

$x_1 + 9 = 10$

$x_1 = 1$

Adding (2), (4) and (6)

$2(y_1 + y_2 + y_3) = 12$

$y_1 + y_2 + y_3 = 6$

$4 + y_3 = 6$

$y_3 = 2$

from(4)

$y_2 + 2 = -4$

$y_2 = -6$

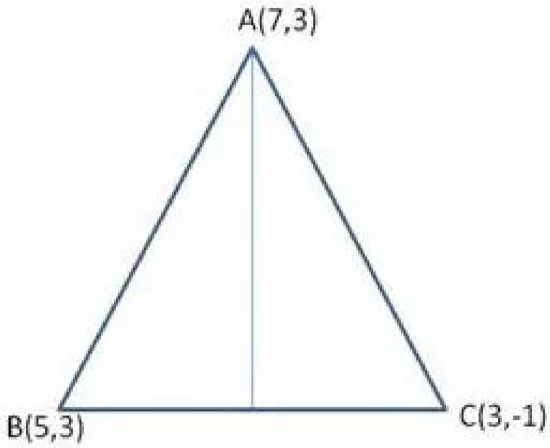
from(6)

$y_1 + 2 = 12$

$y_1 = 10$

The coordinates of the vertices of ΔABC are (9,2), (1,10) and (-7,-6

Answer 17.



we know that the median of triangle bisects the opposite side

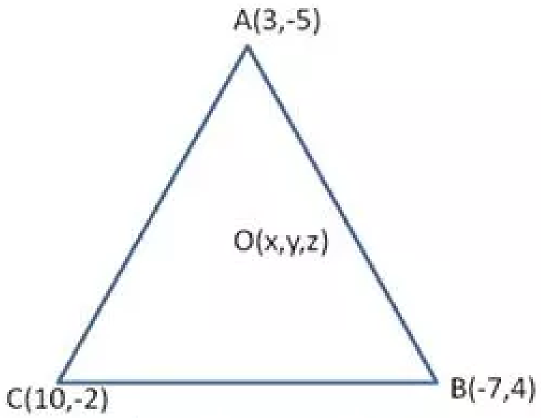
$\therefore$ BD : DC = 1 : 1

Coordinates of D are,

$$D(x, y) = D\left(\frac{5+3}{2}, \frac{3-1}{2}\right) = D(4, 1)$$

Length of median AD $= \sqrt{(7-4)^2 + (-3-1)^2} = \sqrt{9+16} = \sqrt{25} = 5$ units

Answer 18.



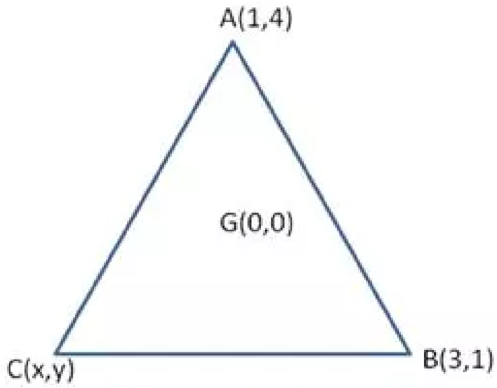
Let O be he centroid of ΔABC.

Coordinates of O are

$$O(x, y, z) = O\left(\frac{3+10-7}{3}, \frac{-5+4-2}{3}\right)$$

$$= O(2,-1)$$

Answer 19.



Given the centroid of ΔABC is at origin, i.e. G(0,0).
Let the coordinates of third vertex be (x,y).
Coordinates of G are,

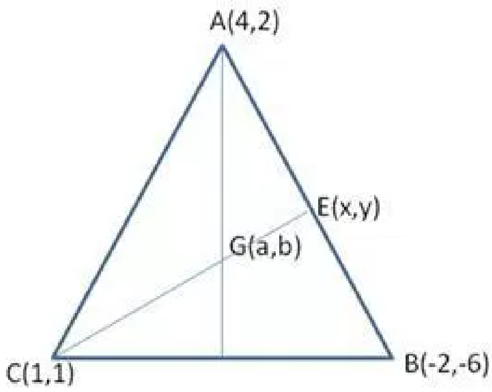
$$G(0,0) = G\left(\frac{1+3+x}{3}, \frac{4+1+y}{3}\right)$$

$$0 = \frac{4+x}{2}, 0 = \frac{5+y}{2}$$

$$x = -4, y = -5$$

Coordinates of third vertex are (-4,-5)

Answer 20.



let G(a,b) be at centroid of ΔABC,
Coordinates of G are,

$$G(a,b) = G\left(\frac{4-2+1}{3}, \frac{2-6+1}{3}\right) = G(1,-1)$$

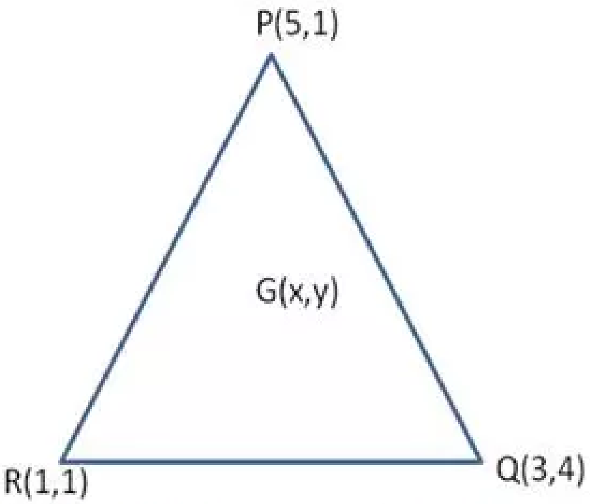
Let CE be the median through C
$\therefore$ AE : EB = 1 : 1
Coordinates of E are

$$E(x,y) = E\left(\frac{4-2}{2}, \frac{2-6}{2}\right) = E(1,-2)$$

Length of median CE

$$= \sqrt{(1-1)^2 + (2-1)^2}$$
$$= \sqrt{9}$$
$$= 3\text{units}$$

Answer 21.



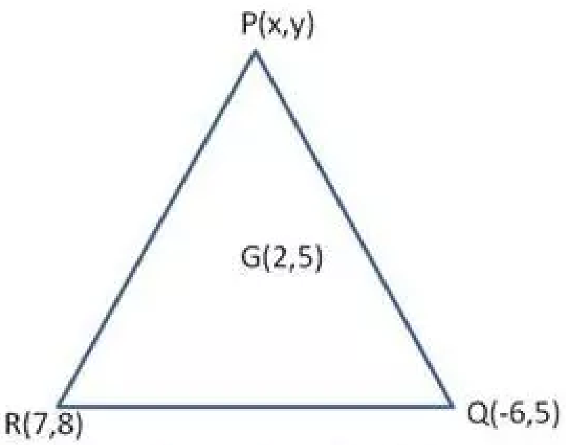
let G(x,y) be the centroid of ΔPQR

Coordinates of G are,

$$G(x,y) = G\left(\frac{5+3+1}{3}, \frac{1+4+1}{3}\right)$$

$$= G(3,2)$$

Answer 22.



Let G be the centroid of ΔPQR whose coordinates are (2,5) and let (x,y) be the coordinates of vertex P.

Coordinates of G are,

$$G(2,5) = G\left(\frac{x-6+7}{3}, \frac{y+5+8}{3}\right)$$

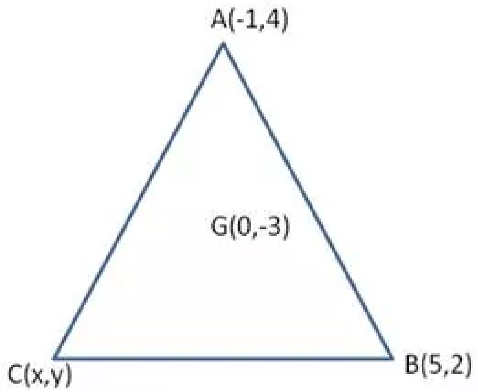
$$2 = \frac{x+1}{3}, 5 = \frac{y+13}{3}$$

$6 = x + 1, \quad 15 = y + 13$

$x = 5, \quad y = 2$

Coordinates of vertex P are (5,2)

Answer 23.



Let G be the centroid of ΔABC whose coordinates are (0,-3) and let C(x,y) be the coordinates of third vertex

Coordinates of G are,

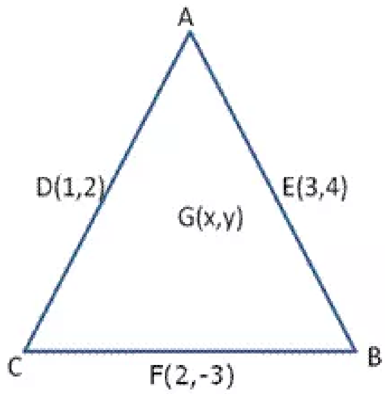
$$G(0,-3) = G\left(\frac{-1+5+x}{3}, \frac{4+2+y}{3}\right)$$

$$0 = \frac{4+x}{3}, -3 = \frac{6+4}{3}$$

x = -4, y = -15

Coordinates of third vertex are (-4,-15)

Answer 24.



Let ABC be a triangle

The midpoint of whose sides AC, AB and BC are D, E and F respectively.

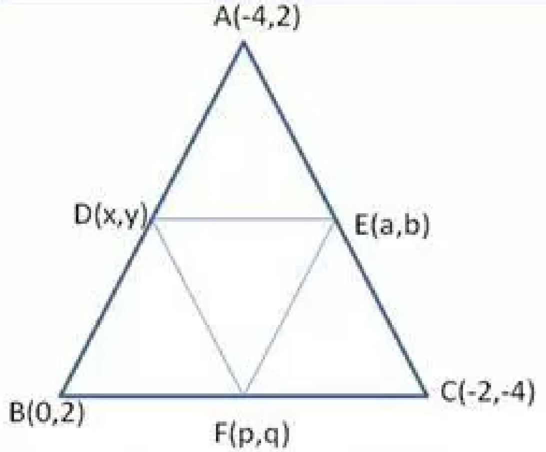
We know that the centroid of ΔDEF. Let G(x,y) be the centroid of ΔABC and ΔDEF

Coordinates of centroid G are,

$$G(x,y) = G\left(\frac{1+3+2}{3}, \frac{2+4-3}{3}\right)$$

$$= G(2,1)$$

Answer 25.



Let D, E and F be the midpoints of the sides AB, AC and BC of ΔABC respectively.

$\therefore$ AD : DB = BF : FC = AE : EC = 1 : 1

Coordinates of D are,

$$D(x, y) = D\left(\frac{0-4}{2}, \frac{2+2}{3}\right) = D(-2, 2)$$

Similarly,

$$E(a, b) = E\left(\frac{-4-2}{2}, \frac{2-4}{2}\right) = E(-3, -1)$$

and,

$$F(p, q) = F\left(\frac{0-2}{2}, \frac{2-4}{2}\right) = F(-1, -1)$$

Coordinates of centroid of ΔABC are,

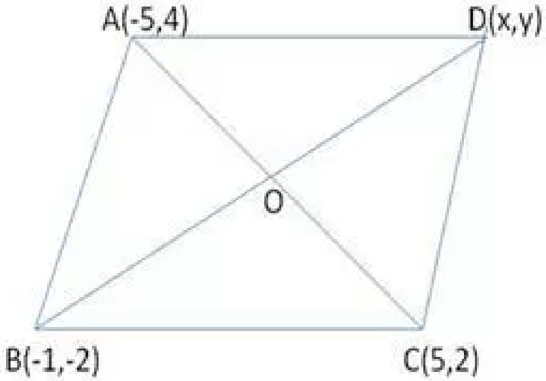
$$= \left(\frac{-4-2+0}{3}, \frac{2-4+2}{3}\right) = (-2, 0)$$

Coordinates of centroid of ΔDEF are,

$$= \left(\frac{-2-3-1}{3}, \frac{2-1-1}{3}\right) = (-2, 0)$$

Thus, the centroid of ΔABC and ΔDEF coincides with the centroid of ΔDEF

Answer 26.



$AB = \sqrt{(-1+5)^2 + (-2-4)^2} = \sqrt{16+36} = \sqrt{52}$units

$BC = \sqrt{(-1-5)^2 + (-2-2)^2} = \sqrt{36+16} = \sqrt{52}$units

$AC = \sqrt{(5+5)^2 + (2-4)^2} = \sqrt{100+4} = \sqrt{104}$

$AB^2 + BC^2 = 52 + 52 = 104$

$AC^2 = 104$

$\because$ AB = AC and $AB^2 + BC^2 = AC^2$

$\therefore$ ABC is an isosceles right angled triangle.

Let the coordinates of D be (x,y)

If ABCS is a square,

Midpoint of AC = mid point of BD

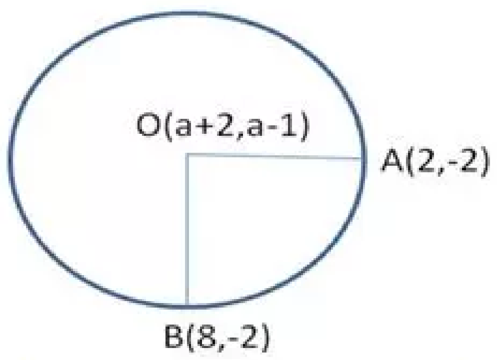
$$O\left(\frac{-5+5}{2}, \frac{4+2}{2}\right) = O\left(\frac{x-1}{2}, \frac{y-2}{2}\right)$$

$$O = \frac{x-1}{2}, 3 = \frac{y-2}{2}$$

x = 1, y = 8

Coordinates of D are (1,8)

Answer 27.

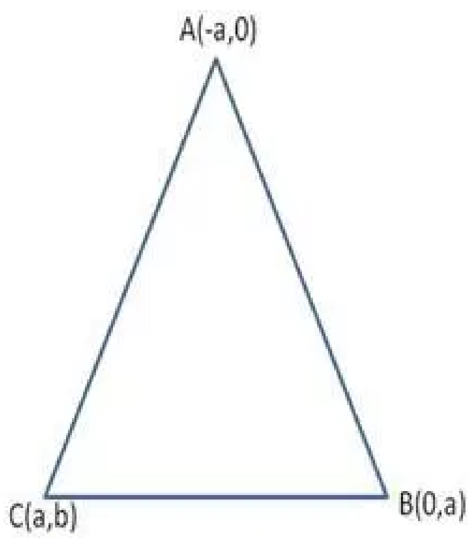


OA = OB [radii of same circle]

$$\therefore OA^2 = OB^2$$
$$(a+2-2)^2 + (a-1+2)^2 = (a+2-8)^2 + (a-1+2)^2$$
$$a^2 + (a+1)^2 = (a-6)^2 + (a+1)^2$$

$$a^2 = a^2 + 36 - 12a$$
$$12a = 36$$
$$a = 3$$

Answer 28.



Coordinates of G are,

$$G(x,y) = G\left(\frac{-a+0+a}{3}, \frac{0+a+b}{3}\right) = G\left(0, \frac{a+b}{3}\right)$$

$$GA^2 = (0+a)^2 + \left(\frac{a+b}{3} - 0\right)^2$$
$$GA^2 = \frac{9a^2 + a^2 + b^2 + 2ab}{9} = \frac{10a^2 + b^2 + 2ab}{9}$$

$$GB^2 = (0-0)^2 + \left(\frac{a+b}{3} - a\right)^2$$

$$GB^2 = \left(\frac{b-2a}{3}\right)^2 = \frac{b^2 + 4a^2 - 4ab}{9}$$

$$GC^2 = (0-a)^2 + \left(\frac{a+b}{3} - b\right)^2$$

$$GC^2 = a^2 + \left(\frac{a-2b}{3}\right)^2 = \frac{9a^2 + a^2 + 4b^2 - 4ab}{9}$$

$$GA^2 + GB^2 + GC^2 = \frac{10a^2 + b^2 + 2ab + b^2 + 4a^2 - 4ab + 10a^2 + 4b^2 - 4ab}{9}$$

$$= \frac{24a^2 + 6b^2 - 6ab}{9}$$

$$GA^2 + GB^2 + GC^2 = \frac{1}{3}(8a^2 + 2b^2 - 2ab) - - - (1)$$

$$AB^2 = (-a-0)^2 + (0-a)^2 = 2a^2$$

$$BC^2 = (0-a)^2 + (a-b)^2 = a^2 + a^2 + b^2 - 2ab = 2a^2 + b^2 - 2ab$$

$$AC^2 = (-a-a)^2 + (0-b)^2 = 4a^2 + b^2$$

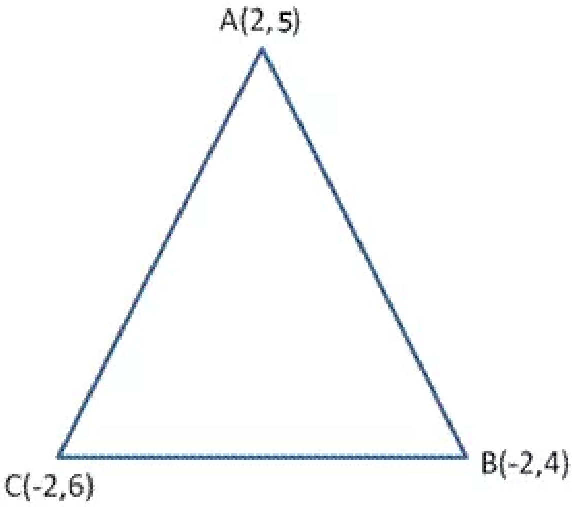
$$AB^2 + BC^2 + AC^2 = 2a^2 + 2a^2 + b^2 - 2ab + 4a^2 + b^2$$

$$AB^2 + BC^2 + AC^2 = 8a^2 + 2b^2 - 2ab - - - (2)$$

from (1) and (2)

$$GA^2 + GB^2 + GC^2 = \frac{1}{3}(AB^2 + BC^2 + AC^2)$$

Answer 29.



$$AB = \sqrt{(2+2)^2 + (5-4)^2} = \sqrt{16+1} = \sqrt{17} \text{ units}$$

$$BC = \sqrt{(-2+2)^2 + (4-6)^2} = \sqrt{0+4} = 2 \text{ units}$$

$$AC = \sqrt{(2+2)^2 + (5-6)^2} = \sqrt{16+1} = \sqrt{17} \text{ units}$$

It can be seen that AB = AC.

Hence, the given coordinates are the vertices of an isosceles triangle.